"Mircea cel Batran" Naval Academy Scientific Bulletin, Volume XVIII – 2015 – Issue 2
Published by "Mircea cel Batran" Naval Academy Press, Constanta, Romania // The journal is indexed in:
PROQUEST SciTech Journals, PROQUEST Engineering Journals, PROQUEST Illustrata: Technology, PROQUEST Technology Journals, PROQUEST Military Collection PROQUEST Advanced Technologies & Aerospace

AN EXTENSION OF THE BINOMIAL MODEL FOR THE MACHINE INTERFERENCE PROBLEM

G. GUREVICH[1]
B. KEREN[2]
Y. HADAD[3]
[1, 2, 3] SCE - Shamoon College of Engineering, Industrial Engineering and Management Department, Beer Sheva, Israel

Abstract: *In this paper an extension of the binomial model for the machine interference problem (MIP) is presented. A production system has several groups of identical machines. All the machines produce the same product and randomly request a service that is provided by a group of operators. Each group of machines has a different priority. The queue discipline is such that machines are served according to their priority (preemptive priority). The model enables calculation of the interference rate for each machine, depending on the number of operators and the priority.*
Keywords: *machine interference problem (MIP), priority, binomial model, queuing.*

INTRODUCTION

The latest version of the binomial model was recently developed by Hadad et al. [1], Keren at al. [2] and Gurevich et al. [3], [4]. This model is applicable for a production system with several machines that produce the same product in parallel and independently of each other. The machines are classified into J $(1 \le J < \infty)$ different groups by types. Each group j, $j = 1,..,J$, includes N_j identical machines. Note that a production system with several types of heterogeneous machines is common in many industries because new machines are frequently introduced either through expansion or replacement. Because the complete elimination of old machines takes years, enterprises are quite likely to have a heterogeneous mixture of both old and new machines (see e.g. [5]). These machines can be expected to differ in run time for production of one unit of a product, failure rate, service time, operation cost, proportion of defects, and similar metrics. Each machine in group j needs a determinable production time T_j (run time) to produce one unit of a product. During the production process each machine may request a service for loading or unloading and for troubleshooting, where the requests are independent. The time that an operator invests in service throughout a cycle is a random variable with an average value of t_j. The assumption is that the value t_j is identical for all the machines in group j. The service is given by the group of K certified operators $\left(1 \le K \le \sum_{j=1}^{J} N_j\right)$ who are qualified to repair all machines. If the number of operators is less than the number of machines, $K < \sum_{j=1}^{J} N_j$, then some machines may wait for a service while the operators give service to other machines (machine interference). The average time that a machine in group j produces one unit of a product is called the cycle time H_j, $j = 1,..,J$. Thus, the cycle time H_j is the sum of three components: the run-time T_j, the average service time t_j, and the average interference time t_{I_j}, i.e., $H_j = T_j + t_j + t_{I_j}$. During waiting time and service time the machines are idle. Thus, in the steady state situation each machine in group j $\left(j = 1,..,J\right)$ can be in one of the following states: running (producing items) with some probability or idle with some probability. These probabilities depend on the average interference time. If the values of these probabilities were known, one could calculate the probability that n_j machines in group j are getting or requiring a service according to the binomial distribution where $n_j = 0,1,...,N_J$. The presented model allows calculation of the average interference time t_{I_j} $\left(j = 1,..,J\right)$ for the machines in each group j where the number of operators K and the priority of each group are given. The value of t_{I_j} depends on the number of operators K, the number of machines in each group N_j, and on queue discipline. The values of T_j, t_j do not depend on K, N_j or on queue discipline. Therefore, T_j, t_j can be evaluated by work measurement (see e.g. [6], [7]). The average interference time enables one to calculate the cycle time H_j, $j=1,..,J$, and steady state probabilities. Using the cycle time H_j, $j=1,..,J$, one can determine an optimal number of operators in the context of different objective functions, for example, to minimize of total manufacturing cost per unit of a product.

MODEL DESCRIPTION

This section presents the model assumptions, notations and application.

2.1 MODEL ASSUMPTIONS

1) There are J $(1 \le J < \infty)$ groups of machines, each group j, $j=1,..,J$, includes N_j identical machines.
2) Each machine of the group j, $j=1,..,J$, can be in one of the following positions, where the probabilities for each position are constant in a steady state situation and identical for all machines of the group:
   a. running (producing items),
   b. having a service,
   c. waiting for a service (interference).
3) Machine failures are independent.
4) Service time is random and each service request transfers immediately to operators.
5) An available operator handles a service request immediately.
6) Each service request is handled by only one operator.
7) Walking time from one machine to another is negligible.
8) A machine is idle while waiting for a service or while getting a service.
9) Each group of machines has a different priority and groups of machines are ranked according to their priorities. Queue discipline serves the machines according to their priority. If an operator must select which of several machines from the same group must be served, that selection is made randomly.
10) Machines are served according to the absolute priority policy (a preemptive priority). When all the operators are busy and an additional machine with a higher priority requests a service, the service of one of the machines with the lowest priority ceases immediately and its operator serves the machine with the higher priority. When a previously interrupted service is resumed, this service is resumed from the point where it was preempted, without loss of the prior work.

2.2 NOTATIONS

N_j-Number of identical machines in the group j, $j=1,...,J$.

T_j- Runtime. The length of time needed for a machine in the group j, $j=1,...,J$, to process one unit of a product. The run time T_j is a pre-given deterministic value.

t_j- Average time of the service that operators invest in a machine of the group j, $j=1,...,J$, during its cycle time (H_j).

t_{I_j}- Interference time. The average time during a cycle time (H_j) during which a machine in the group j, $j=1,...,J$, is idle because it is waiting for an operator.

i_j- Interference proportion. The ratio between the interference time t_{I_j} and the cycle time H_j, that is, $i_j = t_{I_j} / H_j$, $j=1,...,J$.

Thus, the cycle time H_j is calculated as follows:

$$H_j = T_j + t_j + t_{I_j} = T_j + t_j + i_j \times H_j = \frac{T_j + t_j}{\left(1-i_j\right)}$$

.

S_j- Service proportion. The ratio between the average time of the service for a machine in group j, t_j, and the cycle time H_j, i.e., $S_j = t_j / H_j$.

p_{j0}- Probability that a machine in group j is running, $j=1,...,J$. This probability is calculated as follows:

$$p_{j0} = \frac{T_j}{H_j} = \frac{T_j\left(1-i_j\right)}{T_j + t_j}.$$

p_j- Probability that a machine in group j is getting or requiring a service (idle), $j=1,...,J$. This probability is calculated as follows:

$p_j = S_j + i_j$, $j=1,...,J$.

Each machine in each group can be in one of two states - running or idle. A machine in the idle state can be in one of two positions - getting the service or waiting for the service. Because these states are mutually exclusive, it is clear that for any machine in group j the follows equality holds: $p_{j0} + p_j = 1$, $j = 1,...,J$.

X_{j0} - Number of running machines in group j (a random variable), $j = 1,...,J$.

X_j - Number of machines in group j that are getting or requiring a service (a random variable), $j = 1,...,J$.

It is clear that $X_{j0} + X_j = N_j$, i.e., the sum of the number of running machines and the number of idle machines of group j, is equal to the number of machines in the group j, $j = 1,...,J$. Because each group has N_j identical machines, the probability p_j is equal in the steady state for all machines in group j. Therefore, one can calculate the probability that $X_j = x_j$, i.e., x_j ($x_j = 0,1,...,N_j$) machines in group j are getting or requiring a service according to the binomial distribution, $X_j \sim Bin\left(p_j, N_j\right)$, $j = 1,...,J$.

K - Number of certified operators, $1 \le K \le \sum_{j=1}^{J} N_j$.

CALCULATION OF THE INTERFERENCE PROPORTION FOR EACH GROUP OF MACHINES

Denote the group with the highest priority as "group 1" ($j = 1$), and so on. Let us L_j be a number of machines in group j that are waiting for the service, $j = 1,...,J$. If the value of expectation $E\left(L_j\right)$ was known, one could calculate the interference proportion as $i_j = E\left(L_j\right)/N_j$, $j = 1,...,J$ (see e.g. [1], [8]). The value of $E\left(L_j\right)$, $j = 1,...,J$, is calculated as follows: for the group with highest priority $(j = 1)$:

$$E\left(L_1\right) = \sum_{m=K+1}^{N_1} P\left(X_1 = m\right) \times \left(m - K\right) = \sum_{m=K+1}^{N_1} \binom{N_1}{m} \left(p_1\right)^m \left(1 - p_1\right)^{N_1 - m} \times \left(m - K\right), \qquad ((1)$$

and for the other groups:

$$E\left(L_j\right) = E\left(E\left(L_j \middle| \sum_{i=1}^{j-1} X_i\right)\right), \quad j = 2,...,J. \qquad ((2)$$

The random variables X_j and $\sum_{i=1}^{j-1} X_i$, $j = 2,...,J$, are independent. Therefore,

$$E\left(L_j \middle| \sum_{i=1}^{j-1} X_i\right) = \sum_{m=max\left\{0, K+1-\sum_{i=1}^{j-1} X_i\right\}}^{N_j} P\left(X_j = m\right) \times \left(m - max\left\{0, K - \sum_{i=1}^{j-1} X_i\right\}\right)$$
$$= \sum_{m=max\left\{0, K+1-\sum_{i=1}^{j-1} X_i\right\}}^{N_j} \binom{N_j}{m} \left(p_j\right)^m \left(1 - p_j\right)^{N_j - m} \times \left(m - max\left\{0, K - \sum_{i=1}^{j-1} X_i\right\}\right). \qquad ((3)$$

By (2) and (3),

$$E\left(L_j\right) = \sum_{n=0}^{\sum_{i=1}^{j-1} N_i} \sum_{m=max\{0, K+1-n\}}^{N_j} \left(\binom{N_j}{m} \left(p_j\right)^m \left(1 - p_j\right)^{N_j - m} \times \left(m - max\{0, K - n\}\right) \times P\left(\sum_{i=1}^{j-1} X_i = n\right)\right), j = 2,...,J. \qquad ((4)$$

Note that $\sum_{i=1}^{j-1} X_i$ is a sum of independent binomial random variables, i.e., $X_i \sim Bin\left(N_i, p_i\right)$, $i = 1,..., j-1$. For a special case where $J = 2$ the probability is:

$$P\left(\sum_{i=1}^{j-1} X_i = n\right) = P\left(X_1 = n\right) = \binom{N_1}{n} \left(p_1\right)^n \left(1 - p_1\right)^{N_1 - n}, \quad n = 0,1,...,N_1, \qquad K(5)j$$

for $J = 3$ the probability of the sum of two independent binomial random variables is:

$$P\left(\sum_{i=1}^{2} X_i = n\right) = P\left(X_1 + X_2 = n\right) = \sum_{i=0}^{N_1} P\left(X_1 = i\right) \times P\left(X_2 = n - i\right)$$
$$= \sum_{i=0}^{N_1} \left(\binom{N_1}{i} \left(p_1\right)^i \left(1 - p_1\right)^{N_1 - i} \times \binom{N_2}{n-i} \left(p_2\right)^{n-i} \left(1 - p_2\right)^{N_2 - n + i} \times I\left(0 \le n - i \le N_2\right)\right), \qquad ((6)$$

$n = 0,1,...,N_1 + N_2$, where $I\left(\cdot\right)$ is the indicator function. Similarly for $J > 3$ one can calculate the distribution of $\sum_{i=1}^{j-1} X_i$ recursively, by finding the distribution of $\sum_{i=1}^{j-2} X_i$ and then adding the remaining X_{j-1} as is presented by the following equation :

$$P\left(\sum_{i=1}^{j-1} X_i = n\right) = P\left(\sum_{i=1}^{j-2} X_i + X_{j-1} = n\right) = \sum_{k=0}^{\sum_{i=1}^{j-2} N_i} P\left(\sum_{i=1}^{j-2} X_i = k\right) \times P\left(X_{j-1} = n - k\right). \qquad ((7)$$

Thus, with modern computing tools, given the values of N_i and p_i, $i = 1,..., j-1$, it is possible to calculate the exact distribution of

$\sum_{i=1}^{j-1} X_i$, $j = 2,...,J$. The interference proportion i_1 of machines of the group 1 ($j=1$) is calculated as the solution of the following equation:

$$i_1 = \frac{E(L_1)}{N_1}. \qquad ((8)$$

By substituting $E(L_1)$ from equation (1) into equation (8) and setting $p_1 = i_1 + S_1$, equation (8) has the following form:

$$i_1 = \frac{1}{N_1} \sum_{m=K+1}^{N_1} \binom{N_1}{m} (i_1 + S_1)^m \left(1-(i_1+S_1)\right)^{N_1-m} \times (m-K). \qquad ((9)$$

Hadad et al. [1] showed that equation (9) has a unique feasible solution. Similarly, sequentially for $j = 2,...,J$, i_j is the solution of the follows equation:

$$i_j = \frac{E(L_j)}{N_j}, \qquad ((10)$$

where $p_r = i_r + S_r$, $r = 1,..., j-1$. By substituting $E(L_j)$ from equation (4) into equation (10) and setting $p_j = i_j + S_j$, equation (10) has the form of:

$$i_j = \frac{1}{N_j} \times \sum_{n=0}^{\sum_{i=1}^{j-1} N_i} \sum_{m=max\{0,K+1-n\}}^{N_j} \left(\binom{N_j}{m} (i_j+S_j)^m \left(1-(i_j+S_j)\right)^{N_j-m} \times \left(m - max\{0,K-n\}\right) \times P\left(\sum_{i=1}^{j-1} X_i = n\right) \right). \qquad (11)$$

Because $\sum_{i=1}^{j-1} X_i$ is a sum of independent binomial random variables, where $X_i \sim Bin(N_i, p_i)$, the probability $P\left(\sum_{i=1}^{j-1} X_i = n\right)$, $n = 0,...,\sum_{i=1}^{j-1} N_i$, is completely defined by the probabilities p_r, $r = 1,..., j-1$. Therefore, equation (11) has a single variable i_j. By a similar way as was presented in Hadad et al. [1], equation (11) has a unique feasible solution. The solution can be obtained numerically (e.g., by the interval halving method) or by software tools such as Excel-Solver.

CONCLUSIONS

This paper deals with a special case of the machine interference problem where a production system has several types of machines, and where all the machines work in parallel and produce the same product. The heterogeneous machines are divided into several groups, where each group that includes several identical machines has a different priority. The machines request only one type of service that is provided by a group of operators. The presented model enables calculation of the expected number of machines that are waiting for service in each group of machines and the interference proportion for each group via the binomial probability function.

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