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# Mathematical Model of Nonlinear Ship-Wave Interactions

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**Abstract:** An Indirect Boundary Integral Method is used to solve transient nonlinear ship wave interaction problem. A resulting mixed boundary value problem is solved at each time step using mixed Eulerian-Lagrangian time integration technique. A ship hull form is generated in parametric space using a 13-spline surface representation. The convergence rate of both matrix iteration tehniques are impoved with specially preconditioners. Numerical exemples of mode placements on tipical hull cross section using both techniques are descussed and showed and fully nonlinear ship wave interaction and wave resistance computation are presented.

**1.Introduction**

The purpose of this paper is to examine the transitive problem between the ship and the waves considering an adaptive algorithm on the continuously changing surface of the ship's hull, to which the kinematic boundary condition of the ship's hull must be applied It is known that the transitive problems between the ship and waves in their exact form are difficult to solve due to the non-linearity of the boundary conditions of the free surface and the surface of the ship's hull which is continuously changing.

A mixed Euler-Lagrange scheme is used to solve the nonlinear problem. The basis of the method is the use of time-dependent boundary condition of the free surface..

At each time step, the positions of the free surface and the surface of the body are known.

The potential value on the free surface(Dirichlet condition) and the normal derivative value in the body surface (Neumann condition) are known. This well-defined mixed boundary value problem can be solved using a variety of numerical methods.

The boundary conditions of the kinematic and dynamic surface are then repeated for the next time step. The indirect de-singularized limit integral method is used to solve the mixed limit value problem. The main advantage of this technique is that singular points are located outside the fluid domain. Thus, the need to evaluate singular integrals over the body and free surface is avoided, allowing the use of simple numerical quadratures. As a result, the total computational effort at each time step is greatly reduced. Beck, Cao and Lee [3] and Beck, Cao, Scorpio and Schultz [4] used a Wigley hull as a form of the body for the mathematical representation. This allowed the efficient calculation of coordinates of the shape plan of the ship's hull. In addition, the bow and stern profiles are vertical so that the calculation points could be fixed along the length. Since the ship lenghts do not change when the body’s waterline changes, there is no need to adjust the distance between calculation points.

For previous calculations of the Wigley-type ship’s hull, a cosine distance between nodes in the longitudinal direction and a constant distance in vertical direction were used. There were an equal number of nodes in each calculation point (see fig. 1)

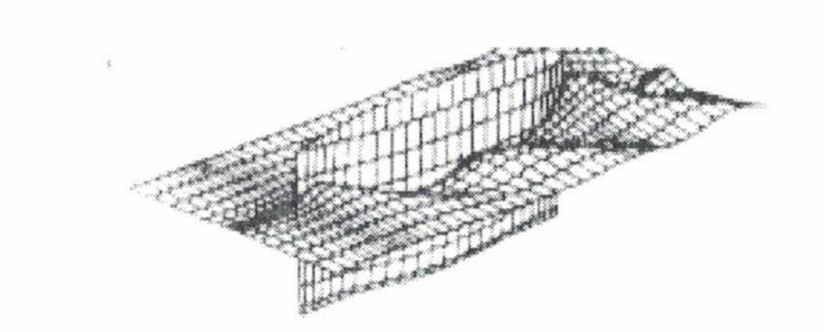

Fig. 1. The first calculations of the Wigley body

For arbitrary hulls, several differences should be considered compared to the calculations for the Wigley body shape. Celebi and Beck studied the nonlinear problem of the ship-wave resistance for the inclined bow and stern of the Wigley body, using adaptive geometric modelling. We will extend this study here by incorporating a new adaptive technique for generating a the grid developed by Vinokur and the iterative calculation algorithms for the block equations matrix.

The second boundary value problem for $\partial \varphi / \partial$ t is established to accurately estimate the accelerations of liquid particles around the body. First, to explain the form arbitrary shape of the ship's hull we will use a parametric representation of the surface of the ship's hull with B-frame, followed by a special transformation to transition from the parameter space to physical space. Secondly, the transverse perimeter of the ship from the keel line to the keel line float can vary significantly in the longitudinal direction. Therefore, it is not desirable to have the same number of nodes on each calculation point. Thirdly, there is a trim(angle of trim) of the bow and stern such that the length of the waterline varies as the body enters and exists the water. To maintain a relatively constant distance between calculation points, the longitudinal positions along the hull’s length, which continuously change, must vary with time. Finally, the distribution of nodes must be sensitive to the shape of the calculation points (for a given number of nodes, the nodes must be concentrated in the areas of curvature big).

**2. The mathematical model**

It is assumed that the liquid is not viscous and the flow is irrational so that the potential of speed exists in the liquid domain. The surface tension is neglected on the free surface.

A cartesian coordinate system 0xyz is chosen so that the plane z = 0 corresponds to the level of calm water and with z is the positive vertical. The coordinate system 0xyz for forward velocity problems, imposes a translation motion of the body in the negative x directon relative to a frame fixed in space. For zero forward speed problems, the body is stationary, and the x-z plane coincides with the central plane of the body. The total velocity potential can be expressed as follows:

$$\Phi = U_0(t)x + \varphi(x, y, z, t) \qquad (1)$$

where: $U_0(t)$ is the velocity of the liquid and $\varphi(x, y, z, t)$ is the disturbance potential. The boundary value problem is governed by the Laplace equation and both $\Phi$ and $\phi$ satisfy the Laplace equation.

The liquid domain is delimited at the top, by the free surface $S_F$, inside of the hull of the ship $S_H$, at the bottom by the flat bottom surface $S_B$ and by the surrounding contour surface $S_\infty$ that includes the upstream and lateral limits.

The most common approach to the time course of boundary conditions for the surface free is the material-node approach in which nodes or collocation points follow individual liquid particles.

Another alternative technique is to prescribe the horizontal displacement of the node and allows node to follow the vertical movement of the free surface. The prescribed displacement of the nodes can be zero so that the positions of the nodes remain fixed in the x-y plane. Depending on the problem, one of these techniques may be easier to apply than another. It is convenient to rearrange the kinematic and dynamic boundary conditions in terms of the time derivative of a point that moves with a predetermined speed v relative to the 0xyz coordinate system. Both conditions limits of the free surface can be put in the form:

$$\frac{\delta\eta}{\delta t} = -(\nabla\varphi - \bar{v})\nabla\frac{\partial\phi}{\partial z} - U_0(t)\frac{\partial\eta}{\partial x} \tag{2}$$

and

$$\frac{\delta\varphi}{\delta t} - g\eta - \frac{1}{2}|\nabla\varphi|^2 + \bar{v}\nabla\varphi - U_0(t)\frac{\partial\varphi}{\partial x} - \frac{p_a}{\delta} \tag{3}$$

where

$$\frac{\delta}{\delta t} = -\frac{\partial}{\partial t} + \bar{v}\nabla \tag{4}$$

is the time derivative that follows the moving node.

If v is equal to $\left(U(t), V(t), \frac{\delta\eta}{\delta t}\right)$ the node follows a prescribed path depending on the speed $(U(t), V(t))$ in the x-y plane and moves vertical to the free surface. The above equations reduce to:

$$\frac{\delta\eta}{\delta t} = -\nabla\varphi\nabla\eta + U_0(t)\frac{\partial\eta}{\partial x} + V(t)\frac{\partial\eta}{\partial y} + \frac{\partial\varphi}{\partial z} - U_0(t)\frac{\partial\eta}{\partial x} \tag{5}$$

and
$$\frac{\delta\varphi}{\delta t} = -g\eta - \frac{1}{2}|\nabla\varphi|^2 + U(t)\frac{\partial\varphi}{\partial x} + V(t)\frac{\partial\varphi}{\partial y} + \frac{\delta\eta}{\delta t}\cdot\frac{\partial\varphi}{\partial z} - U_0(t)\frac{\partial\varphi}{\partial x} - \frac{p_a}{\delta} \tag{6}$$

Establishing the velocity of the liquid $U_0(t) = 0$ we obtain:

$$\frac{\delta\eta}{\delta t} = -\nabla\varphi\nabla\eta + U_0(t)\frac{\partial\eta}{\partial x} + V(t)\frac{\partial\eta}{\partial y} + \frac{\partial\varphi}{\partial z} \tag{7}$$

$$\frac{\delta\varphi}{\delta t} = -g\eta - \frac{1}{2}|\nabla\varphi|^2 + U(t)\frac{\partial\varphi}{\partial x} + V(t)\frac{\partial\varphi}{\partial y} + \frac{\delta\eta}{\delta t}\cdot\frac{\partial\varphi}{\partial z} - \frac{p_a}{\delta} \tag{8}$$

The last form of the kinematic and dynamic boundary conditions allows the determination in time of the value of the free elevation and the potential. A difficulty is the assessment of the elevation gradient of the free surface, $\nabla\eta$. Therefore, it must be evaluated numerically but it takes time.

### 3. Calculations of the nonlinear interaction between the ship and the waves

A Wigley hull was used to validate the mathematical method of creating the nodal distribution surface, as the answers can be compared with previous calculations. The Wigley hull is a mathematical shape that has a length-to-bea, ratio of 10, a bea,-to-draft ratio of 1,6 and the following equation for the surface of the ship's hull:

$$y = \pm \frac{B}{2}\left\{1-\left(\frac{2x}{L}\right)^2\right\}\left\{1-\left(\frac{z}{H}\right)^2\right\} \tag{9}$$

where: L - the length of the ship
B - the width/beam of the ship
H - the draft of the ship

There are two Wigley hulls with straight sides, with different profiles fore and aft used to calculate the ship-wave interaction and the free surface waves generated by ship (fig. 2a and 2b).

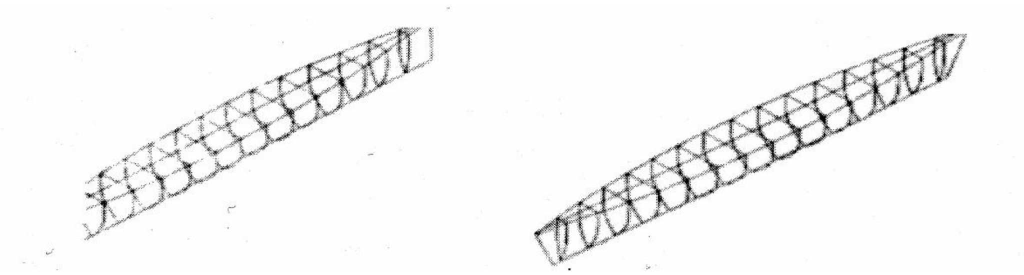

Fig. 2a Standard Wigley body with right board Fig. 2b Wigley body with bow and sloping stern

The second shape of the Wigley body with the inclined profiles at the bow and stern was created from the standard shape of the Wigley body by modifying the bow and stern profiles, the waterlines also corresponding to computing stations. This was achieved by creating a Wigley body using a CAD program and then the bow and stern profiles were reduced below the waterline on the calm sea and magnified above it. The reason for using a frame representation - B for standard forms of the Wigley body was to test node distribution techniques. Then, we can compare the results of frames - B with the previous analytical results of Beck and his collaborators, and the accuracy of the representation of the stiffened surface - B and the distributions is checked nodes. Both models were tested at speeds given by:

$$U_0(t) = U(1-e^{-A}) \tag{10}$$

where A= $0,05t^2$

The reason for choosing a Gaussian start was that it has zero acceleration at t = o, but it can be chosen another formulation of the start such as the hyperbolic tangent. The normalized profiles of the waves of a along the hull of the ship for $t\sqrt{g/L} =$ 20 are given in the figure 3.

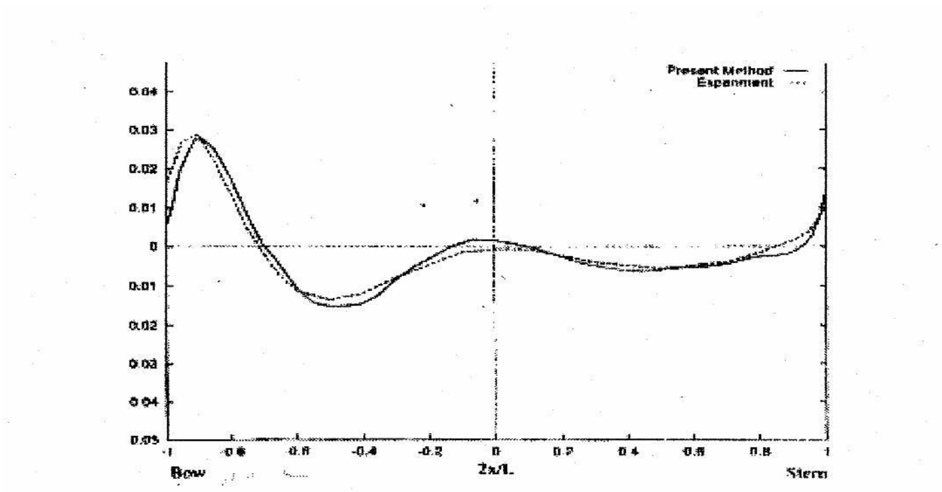


Fig. 3 Comparison of wave profiles along the Wigley body

The experimental result for a standard 2.5m Wigley hull set at immersion and longitudinal inclination (aside) is taken from the work of Noblesse and Mc. Carthy. In general there is a good correspondence with the exception of a small difference and a shift of a the amplitude of the bow accompanying waves. There is a slight over estimation of the master couple and o reduced estimate at bow (2x/L=0,6 $\approx$ -0,4) and stern (2x/L=0,6 $\approx$ 0,9).

Comparisons and the results of the tests showed that the adaptive distribution of the nodes and the approximation of the frame - B a of the Wigley ship's hull are very accurate and acceptable for estimating the wave formation in around the body. The development of the wave system along the plane of symmetry and around of the ship's hull (inclined bow and stern) for Fn=0.289 is shown in figure 4.

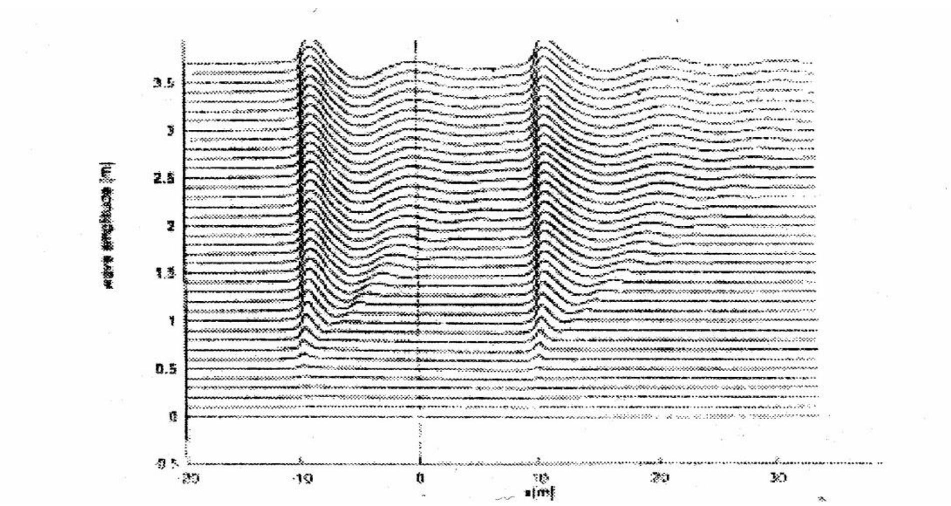


Fig. 4 Development of the wave system along the surface Wigley's body

It can be seen that the amplitude of the wave increases as a function of time and approaches almost constant shape (straight). The wave pattern generated by the Wigley body is investigated in fig. 5 for Fn=0.289 at time $t\sqrt{g/L}=17$. The results indicate that the bow and stern are two basic sources of wave formation. There is a frontal distribution field at the bow and formation transverse waves starts from the bow area due to the forward movement of the body.

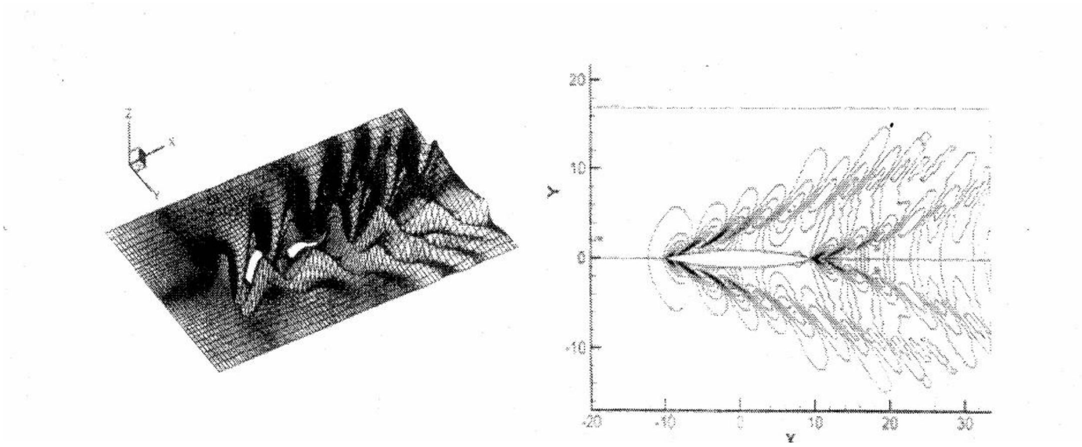

Fig. 5 The waves generated by the ship. Free surface waves for Wigley body with sloping bow and stern

**4. Conclusions**

The indirect de-singularized limit integral method is used for the solution the transitive ship-wave problem in three dimensions. Two adaptive grid generation techniques for ship-wave interaction are proposed and compared. Both techniques redistribute nodal points automatically on the instantly wetted surface of the body at each time step of major integration. Two types of hull shapes are used:

a) Wigley-type body and b) Wigley-type body with inclined bow and stern.

The second shape is created in parametric space using surface approximations frames-B. The nodal points are redistributed by the space weight function or by interpolation transfinite (TFI). The method allows calculations on arbitrary body shapes.

In this study, the shape of the ship's hull is represented by a single surface a of the frames body-B. In general, it is difficult to add a bulbous bow and straight stern (mirror) with the necessary precision. Therefore, for more complex body shapes, the next stage will be the development of the surface representation of the B-frames body, more on that of a surface. The positions of the source points near the bow and stern areas have a direct influence convergence of the iteration process.

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